Atomic detection and matter-waves coherence

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We analyze several models of atomic detectors in the context of the measurement of coherence properties of matter waves. In particular, we show that an ionization scheme measures normally-ordered correlation functions of the Schrödinger field, in analogy with the optical situation. However, it exhibits a sensitivity to exchange processes that is normally absent in optics.

Optical coherence theory is based on the observation that most quantum measurements that can be performed on the electromagnetic field yield a signal proportional to normally ordered correlation functions of that field [1]. A quantized multimode field is then said to be coherent to order N if all normally ordered correlation functions up to order N factorize. No such theory is presently available for atomic coherence, probably because until recently it had not been necessary to think of atomic samples as Schrödinger fields. But the experimental work on ultracold atoms, BEC [2–6] and atom lasers [7] has changed that situation, and the need for a proper theory of atomic coherence is now quite urgent [8].

At least for the case of bosonic fields, it is tempting to simply transpose Glauber's coherence theory [1]. This approach has been the de facto situation so far, but appealing as it might sound, it must be applied with caution, due to the fundamental difference between electromagnetic and matter-wave fields. Most optical experiments detect light by absorption, i.e. by "removing" photons from the light field. This is the reason why normally ordered correlation functions are so important. But atomic detectors work in a number of different ways: One can chose to measure electronic properties, or center-of-mass properties, or both. While one detector might be sensitive to atomic velocities, another might measure local densities and a third electronic properties only. Additional difficulties arise from the fact that atomic fields are self-interacting, which significantly complicates the propagation of atomic coherence as compared to the case of light. From these remarks, it should be clear that a theory of matter waves coherence is much richer than its optical equivalent. Yet, like Glauber's coherence theory, it should be operational and based on explicit detection schemes.

The goal of this note is to analyze several ideal atom detectors and to determine which correlation functions of the matter-wave field they are sensitive to. The systems we explicitly consider are a non-resonant atomic imaging system such as used e.g. in the MIT BEC experiments, and a detector working via atomic ionization. We show that while the off-resonance imaging detector is sensitive to density correlation functions, a narrow-band ionization detector measures normally ordered correlation functions of the Schrödinger field itself, in analogy with the optical case. Intermediate situations are more complicated, due to the quadratic dispersion of matter waves. Higher-order detection schemes also involve exchange terms usually absent in the optical case.

Nonresonant imaging

Consider first atomic detection by non-resonant imaging, a situation where a strongly detuned electromagnetic field interacts with the atoms in the sample in such a way that it induces only virtual transitions. We consider for concreteness ground state atoms described by the Schrödinger field operator $\hat{\Psi}(\mathbf{r})$ with $[\hat{\Psi}(\mathbf{r}), \hat{\Psi}^\dagger(\mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}')$ for bosons, and decompose the electromagnetic field into a classically populated mode of wave vector $\mathbf{k}_0$ and polarization $\boldsymbol{\epsilon}_0$ and a series of weakly excited sidemodes of wave vectors $\mathbf{k}_\ell$ and polarizations $\boldsymbol{\epsilon}_\ell$. After adiabatic elimination of the upper electronic state of the atomic transition under consideration, the interaction between the Schrödinger field and the radiation field is described to lowest order in the side-modes by the effective Hamiltonian

$$
\begin{aligned}
V = {} & \hbar \int d^3 r \frac{|\Omega_0(\mathbf{r})|^2}{\delta_0} \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}(\mathbf{r}) \\
& + \hbar \sum_\ell \int d^3 r \left(\frac{\Omega_0(\mathbf{r})\Omega_\ell^\star}{\delta_0} a_\ell^\dagger e^{i(\mathbf{k}_0 - \mathbf{k}_\ell)\cdot\mathbf{r}} \right. \\
& \left. + \frac{\Omega_0^\star(\mathbf{r})\Omega_\ell}{\delta_0} a_\ell e^{-i(\mathbf{k}_0 - \mathbf{k}_\ell)\cdot\mathbf{r}} \right) \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}(\mathbf{r}), \qquad (1)
\end{aligned}
$$

where $\mathbf{k}_\ell$ is the wave vector of the ℓ-th mode of the field, of frequency ω_ℓ and polarization $\boldsymbol{\epsilon}_\ell$, the sum is over all field modes in the quantization volume V, and $\mathcal{E}_\ell = [\hbar\omega_\ell/2\epsilon_0 V]^{1/2}$ is the "electric field per photon" of mode ℓ. The annihilation and creation operators a_ℓ and $a_\ell^\dagger$ satisfy the boson commutation relation $[a_\ell, a_{\ell'}^\dagger] = \delta_{\ell,\ell'}$. We have also introduced the Rabi frequencies $\Omega_0(\mathbf{r}) = d\mathcal{E}_0(\mathbf{r})(\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}_0)/\hbar$ and $\Omega_\ell = d\mathcal{E}_\ell(\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}_\ell)/\hbar$, and the atom-field detuning $\delta_0 \equiv$

$\omega_a - \omega_0$ is assumed to be much larger than Ω_0, $\delta_0 \gg \Omega_0(\mathbf{r})$.

Assuming that the electromagnetic field is initially in the state $|\mathcal{E}\rangle$ and the Schrödinger field in the state $|\phi_g\rangle$, the probability that the system undergoes a transition from that to another state is given to first order in perturbation theory by

$$w = \frac{1}{\delta_0^2} \sum_{\ell,\ell'} \Omega_\ell^\star \Omega_{\ell'} \int d^3r \Omega_0(\mathbf{r}) \int d^3r' \Omega_0^\star(\mathbf{r}')$$

$$\int_0^{\Delta t} dt \int_0^{\Delta t} dt' \langle \phi_g | \hat{\rho}(\mathbf{r},t) \hat{\rho}(\mathbf{r}',t') | \phi_g \rangle$$

$$\times \langle \mathcal{E} | a_\ell^\dagger a_{\ell'} e^{i(\mathbf{k}_{\ell'} \cdot \mathbf{r}' - \mathbf{k}_\ell \cdot \mathbf{r})} e^{-i(\omega_{\ell'} t' - \omega_\ell t)} + h.c. | \mathcal{E} \rangle \quad (2)$$

where the Schrödinger wave density is defined as

$$\hat{\rho}(\mathbf{r},t) \equiv \hat{\Psi}^\dagger(\mathbf{r},t) \hat{\Psi}(\mathbf{r},t) \quad (3)$$

and $\Psi(\mathbf{r},t) = U^\dagger \Psi(\mathbf{r}) U$ is the time-dependent Schrödinger field in the interaction representation with respect to the atomic Hamiltonian, i.e. $U = \exp(-i\mathcal{H}_A t/\hbar)$.

We further assume for concreteness that all electromagnetic sidemodes are initially in a vacuum. The measurement on the Schrödinger field is then carried out by detecting photons scattered by the atoms into the sidemodes, in a fashion familiar from resonance fluorescence experiments. The most important non-trivial contribution to the fluorescence signal is proportional to the intensity $|\Omega_0|^2$ of the incident field,

$$w = \frac{|\Omega_0|^2}{\delta_0^2} \sum_\ell |\Omega_\ell|^2 \int d^3r \int d^3r' \int_0^{\Delta t} dt \int_0^{\Delta t} dt'$$

$$e^{i(\mathbf{k}_0 - \mathbf{k}_\ell) \cdot (\mathbf{r} - \mathbf{r}')} e^{-i(\omega_0 - \omega_\ell)(t - t')}$$

$$\times \langle \phi_g | \hat{\rho}(\mathbf{r},t) \hat{\rho}(\mathbf{r}',t') | \phi_g \rangle, \quad (4)$$

and hence is sensitive to the second-order correlation function of the sample density. This is to be compared to the results of Javanainen [9], who showed that the the spectrum of the scattered radiation is a function of $\langle \hat{\rho}(\mathbf{r},0)\hat{\rho}(\mathbf{r},t) \rangle$. Indeed, it can be shown in all generality that any measurement involving the electromagnetic field scattered by the atomic sample under conditions of off-resonant imaging are determined by correlation functions of the Schrödinger field density.

Ionization

The reason off-resonant imaging yields a signal dependent on $\hat{\rho}(\mathbf{r},t)$ is that the electric dipole interaction is bilinear in the Schrödinger field operators. This difficulty can however be eliminated if, instead of making measurements on the radiation field, one detects the atoms directly. One scheme that achieves

this goal is the ionization method that we now discuss.

Consider a detector consisting of a tightly focussed laser beam that can ionize atoms by inducing transitions from their ground electronic level $|g\rangle$ to a continuum level $|i\rangle$. [1] The corresponding single-particle Hamiltonian is

$$H = H_{cm} + H_{el} + V(\mathbf{r}) \equiv H_0 + V \quad (5)$$

where H_{cm} is the center-of-mass Hamiltonian, H_{el} the electronic Hamiltonian, and $V(\mathbf{r})$ describes the electric dipole interaction between the atom and the ionizing laser field. H_{el} has eigenstates φ_n and eigenfrequencies ω_n, $H_{el}|\varphi_n\rangle = \hbar\omega_n|\varphi_n\rangle$. The corresponding atomic manybody Hamiltonian is

$$\mathcal{H}_0 = \int d^3r \hat{\Psi}^\dagger(\mathbf{r}) H_0 \hat{\Psi}(\mathbf{r}) \quad (6)$$

where in the Born-Oppenheimer approximation $\hat{\Psi}(\mathbf{r})$ is a multicomponent field with components $\hat{\Psi}_n(\mathbf{r})$.

We are interested in measuring properties of the ground state component $\hat{\Psi}_g(\mathbf{r})$ of this field, which is dipole-coupled to continuum states $\hat{\Psi}_i(\mathbf{r})$. We assume for simplicity that the center-of-mass wave function of these latter states is well described by plane waves of momentum $\mathbf{q}$, so that $\mathcal{H}$ may be expressed as

$$\mathcal{H}_0 = \mathcal{H}_g + \sum_i \mathcal{H}_i, \quad (7)$$

where

$$\mathcal{H}_i = \hbar \sum_{\mathbf{q}} (\omega_q + \omega_i) b_{\mathbf{q},i}^\dagger b_{\mathbf{q},i}. \quad (8)$$

Here we expanded $\hat{\Psi}_i(\mathbf{r})$ in plane waves as $\hat{\Psi}_i(\mathbf{r}) = \sum_{\mathbf{q}} \phi_{i,\mathbf{q}}(\mathbf{r}) b_{\mathbf{q},i}$ with $[b_{\mathbf{q},i}, b_{\mathbf{q}',i'}^\dagger] = \delta_{\mathbf{qq}'}\delta_{ii'}$, and $\omega_q = \hbar q^2/2M$. (Note that the inclusion of ground state collisions is straightforward and does not affect our conclusions.)

In terms of the components $\hat{\Psi}_n(\mathbf{r})$ of the Schrödinger field, the electric dipole interaction Hamiltonian is

$$\mathcal{V} = \hbar \sum_i \int d^3r \Omega_i(\mathbf{r}) \hat{\Psi}_i^\dagger(\mathbf{r}) \hat{\Psi}_g(\mathbf{r}) + H.c., \quad (9)$$

where Ω_i is the Rabi frequency between the levels $|g\rangle$ and $|i\rangle$, and the laser field is treated classically.

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[1] Hot wire detectors can be modeled in a similar fashion.

In this detection scheme, one extracts information about the state of the field $\hat{\Psi}_g(\mathbf{r},t)$ by measuring, e.g. the number of atoms in the continuum. For atoms cooled well below the recoil temperature and tightly focused laser beams, the spatial size of the atomic wave function is much larger than the laser spot and we can approximate the electric field $\mathbf{E}(\mathbf{r})$ by $\mathbf{E}(\mathbf{r}) \simeq \mathbf{E}\delta(\mathbf{r} - \mathbf{r}_0)$, so that Eq. (9) becomes

$$\mathcal{V} = \hbar \sum_i \Omega_i(\mathbf{r}_0)\hat{\Psi}_i^\dagger(\mathbf{r}_0)\hat{\Psi}_g(\mathbf{r}_0) + H.c. \qquad (10)$$

We take the atomic system to be initially in the state

$$|\psi(0)\rangle = |\{\psi_{i,\mathbf{q}}(0)\}, \psi_g(0)\rangle. \qquad (11)$$

To first order in perturbation theory, the transition probability away from that state during the time interval Δt is

$$w = \sum_{i,q} |\Omega_i(\mathbf{r}_0)|^2 \int_0^{\Delta t} dt \int_0^{\Delta t} dt'$$
$$\langle \psi_{i,\mathbf{q}}(0)|\hat{\Psi}_i(\mathbf{r}_0,t)\hat{\Psi}_i^\dagger(\mathbf{r}_0,t')|\psi_{i,\mathbf{q}}(0)\rangle$$
$$\times \langle \psi_g(0)|\hat{\Psi}_g^\dagger(\mathbf{r}_0,t)\hat{\Psi}_g(\mathbf{r}_0,t')|\psi_g(0)\rangle + c.c. \qquad (12)$$

There is a fundamental distinction between the present situation and Glauber's photodetection theory, because in the present case both the detected and detector fields consist of matter waves. There is a complete symmetry between these two fields so far, and their roles are interchangeable. In order to break this symmetry and to truly construct a detector, we now make a series of assumptions on the state of the detector fields $\hat{\Psi}_i(\mathbf{r},t)$. Physically, this amounts to making a statement about the way the detector is prepared prior to a measurement. Specifically, we assume that all atoms are in the ground state, $\Psi_i(\mathbf{r}_0)|\psi_{i,\mathbf{q}}(0)\rangle = |0\rangle$, and that any atom in an ionized state will be removed from the sample instantaneously. In that case, the second term in Eq. (12) vanishes and we have

$$w = \sum_i |\Omega_i(\mathbf{r}_0)|^2 \int_0^{\Delta t} dt \int_0^{\Delta t} dt'$$
$$\sum_q e^{i\omega_q(t-t')}\phi_q(\mathbf{r}_0)\phi_q^\star(\mathbf{r}_0)$$
$$\times e^{i\omega_i(t-t')}\langle \psi_g|\hat{\Psi}_g^\dagger(\mathbf{r}_0,t)\hat{\Psi}_g(\mathbf{r}_0,t')|\psi_g\rangle. \qquad (13)$$

At this point, it is convenient to distinguish three different operating regimes: In the first one, only one final electronic state is considered, and in addition a velocity selector is used to filter just those ionized atoms with a given center-of-mass momentum. We call this a *narrowband single-state detector.* The second scheme allows for a broader velocity filter, but

still considers a single continuum electronic state, and we call it a *broadband single-state detector.* Finally, we also discuss a *general broadband detector* where neither the final momentum state nor the final electronic state is narrowly selected.

More precisely, a narrowband single-state detector includes a velocity selector with a bandwidth $\Delta\mathbf{q}$ around a central value $\mathbf{q}_0$ such that for the detection times Δt of interest, one has $\Delta t \Delta\omega_\mathbf{q} \ll 1$, where $\Delta\omega_\mathbf{q} = \hbar\mathbf{q}_0\Delta\mathbf{q}/2M$. In this case and for a stationary Schrödinger fields Eq.(13) reduces to

$$r_{nb}(\omega, \omega_{\mathbf{q}_0}) = \frac{\Delta\omega_\mathbf{q}^3}{c^3}|\Omega(\mathbf{r}_0)|^2$$
$$\times \int_0^{\Delta t} d\tau e^{-i(\omega+\omega_{\mathbf{q}_0})\tau}G_A(0,\tau;\mathbf{r}_0,\mathbf{r}_0), \quad (14)$$

where we dropped the index i of the observed continuum state for clarity, introduced the ionization rate $r_{nb}(\omega, \omega_\mathbf{q}) = w_{nb}(\omega, \omega_\mathbf{q})/\Delta t$ and defined the atomic normally ordered first-order ground state correlation function

$$G_A(t,t';\mathbf{r}_0,\mathbf{r}_0) = \langle \phi_g|\hat{\Psi}_g^\dagger(\mathbf{r}_0,t)\hat{\Psi}_g(\mathbf{r}_0,t')|\phi_g\rangle.$$

From the Wiener-Khintchine theorem, we recognize that for large enough Δt, the detector measures the spectrum of the Schrödinger field $\hat{\Psi}_g(\mathbf{r}_0,0)$.

In the case of a broad single-state detector, in contrast, we have

$$r_{1b} \simeq |\Omega(\mathbf{r}_0)|^2 \int_0^{\Delta t} d\tau e^{-i\omega\tau}G_{pr}(0,\tau;\mathbf{r}_0,\mathbf{r}_0)$$
$$\times G_A(0,\tau;\mathbf{r}_0,\mathbf{r}_0) \qquad (15)$$

where we have introduced the center-of-mass propagator

$$G_{pr}(t_1,t_2;\mathbf{r}_1,\mathbf{r}_2) = \sum_q \phi_q(\mathbf{r}_1)\phi_q^\star(\mathbf{r}_2)e^{i\omega_q(t_2-t_1)}.$$
$$(16)$$

In that case, the ionization rate is proportional to the Fourier transform of the product of $G_{pr}(0,\tau;\mathbf{r}_0,\mathbf{r}_0)$ and the correlation function $G_A(0,\tau;\mathbf{r}_0,\mathbf{r}_0)$, or in other words to the convolution of the Fourier transforms of these functions. The Fourier transform of the center-of-mass propagator can therefore be interpreted as the spectral resolution of the detector.

We finally turn to the case of a general broadband detector, where the spectrum of the detector is much broader than the spectrum of the detected quantity. Assuming that the spectrum of the atomic correlation function is centered at $\bar{\omega}$, we find

$$r_{bb} \simeq \eta(\mathbf{r}_0)G_A(0,0;\mathbf{r}_0,\mathbf{r}_0), \qquad (17)$$

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where we have introduced the "detector efficiency"

$$\eta(\mathbf{r}_0) = \int d\tau \sum_i |\Omega_i(\mathbf{r}_0)|^2 \langle \Psi_i(\mathbf{r}_0, \tau) \Psi_i^\dagger(\mathbf{r}_0, 0) \rangle e^{-i\bar{\omega}\tau}. \tag{18}$$

As expected, a broadband detector is not able to resolve any spectral feature of the Schrödinger field, and only measures the local atomic density.

Higher-order correlations

The detection of higher-order correlation functions of the Schrödinger field can be achieved by a straightforward generalization of the ionization detector. For instance, second-order coherence measurements can be carried out by focussing the laser at two locations $\mathbf{r}_1$ and $\mathbf{r}_2$, in which case

$$\mathcal{V} = \hbar \sum_{\mu=1,2} \sum_i \Omega_i(\mathbf{r}_\mu) \hat{\Psi}_i^\dagger(\mathbf{r}_\mu) \hat{\Psi}_g(\mathbf{r}_\mu) + H.c. \tag{19}$$

Assuming as before that the continuum states are initially empty and for a general broadband detector, the joint probability to ionize an atom at $\mathbf{r}_1$ and another one at $\mathbf{r}_2$ is then

$$\begin{aligned}
w_2 \simeq {} & \eta(\mathbf{r}_1, \mathbf{r}_2) \eta(\mathbf{r}_2, \mathbf{r}_1) \int_0^{\Delta t} dt_1 \int_0^{\Delta t} dt_2 \\
& \langle \hat{\Psi}_g^\dagger(\mathbf{r}_1, t_1) \hat{\Psi}_g^\dagger(\mathbf{r}_2, t_2) \hat{\Psi}_g(\mathbf{r}_2, t_1) \hat{\Psi}_g(\mathbf{r}_1, t_2) \rangle \\
& + \eta(\mathbf{r}_1) \eta(\mathbf{r}_2) \int_0^{\Delta t} dt_1 \int_0^{\Delta t} dt_2 \\
& \langle \hat{\Psi}_g^\dagger(\mathbf{r}_1, t_1) \hat{\Psi}_g^\dagger(\mathbf{r}_2, t_2) \hat{\Psi}_g(\mathbf{r}_2, t_2) \hat{\Psi}_g(\mathbf{r}_1, t_1) \rangle, \tag{20}
\end{aligned}$$

where we have introduced the detector cross efficiency $\eta(\mathbf{r}_1, \mathbf{r}_2)$ as a straightforward generalization of Eq. (18). The first term in Eq. (20) is an exchange term resulting from the fact that the detector field is a single Schrödinger field. It results from the interference between detectors at points $\mathbf{r}_1$ and $\mathbf{r}_2$. The second term is the usual term also appearing in the double photo-detection of optical fields. In that latter case, the exchange term does not appear because the two detectors used to measure the field are taken to be distinguishable. Note also that in the position measurement scheme proposed in Ref. [10], interferences do not occur as the set of states ionized at each location are taken to be distinguishable. We finally remark that as a consequence of the exchange term, the signal cannot simply be expressed in terms of correlation functions of $\hat{\rho}(\mathbf{r}, t)$.

In summary, we have analyzed several detectors that permit to access different classes of correlation functions of the Schrödinger field. Most interesting perhaps is the ionization scheme, which is closely related to the detectors familiar from the detection of optical fields. However, it presents new features, and is in particular sensitive to exchange processes. But ionization detectors make destructive measurements. This is in contrast to off-resonant imaging, which is nondestructive but measures density correlation functions instead of the more familiar normally-ordered correlation functions of the Schrödinger field itself.

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